International Journal of Scientific & Engineering Research, Volume 6, Issue 7, July-2015
ISSN 2229-5518

More Authentic Proof of Fermat Number that it is composite for n > 4

Author: Debajit Das

ABSTRACT

This is the extended version of my earlier publication in this journal i.e. for the month of June edition 2015. I mentioned there that the proof of Fermat number to be composite beyond n = 4 is fully based on a conjecture that if $u^2 + 1$ is composite, $u^4 + 1$ is also composite. This conjecture was proved by the property of N-equation which was published in August edition 2013. To prove this conjecture I considered the fact that $(u^4 + 1)$ satisfies only the left hand odd element of a second kind N-equation $a^2 + b^2 = c^2$. But $(u^4 + 1)$ may also satisfy the left hand odd element of a first kind N-equation except k = 1. This paper includes the fact that in both the cases the proof remains undisturbed. Because all the left hand odd elements except k = 1 are composite.

Keywords

(a, b)-consecutive phenomenon, N-equation, Mixed zygote form, Natural constant (k), wing

1. Introduction

N-equation $a^2 + b^2 = c^2$ (a, b, c can be said as its elements) is nothing but the systematic arrangement of all Pythagorean triplets. According to this arrangement the N-equation has been divided into two kinds i.e. 1st kind includes where k = c – b is in the form of 1^2, 3^2, 5^2,…….and 2nd kind includes where k = c – b is in the form of 2.1^2, 2.2^2, 2.3^2, …….. assuming a < b < c in both the cases. Regarding prime numbers' distribution if we look its arrangement in 'Mixed zygote' form we will observe that all the prime numbers satisfy the left hand odd element of a N-equation for k = 1 only. All other left hand odd elements except k = 1 are composite. Moreover, for k = 1, the nature of conjugate zygote expression i.e. $(\alpha^2 \pm \beta^2)$ is such that α, β are always consecutive integers.
Moreover, RH odd element ± LH odd element = $2(\text{integer})^2$ and RH odd element ± LH even element = $(\text{odd integer})^2$
From N_s operation we can review the following property.
$\pi\, (e_i^2 + o_i^2) = (e_1^2 + o_1^2)\,(e_2^2 + o_2^2)(e_3^2 + o_3^2)\ldots\ldots\, (e_n^2 + o_n^2) = E(v_j^2 + d_j^2) = (v_1^2 + d_1^2) = (v_2^2 + d_2^2) = (v_3^2 + d_3^2) = \ldots\ldots\ 2^{n-1}$ wings where the symbol π & E stand for continued product & equalities, e, v for even integers & o, d for odd integers. All $(e_i^2 + o_i^2)$ are prime numbers & $\gcd(e_i, o_i) = \gcd(v_j, d_j) = 1$. If any prime is repeated then in all cases $\gcd(v_j, d_j) \neq 1$
Based on the above theoretical background we can prove that the Fermat number ($F_n = 2^{2^{\wedge}n} + 1$) will always represent a composite number for n > 4

1. For a N-equation $a^2 + b^2 = c^2$ how a, b form consecutive integers.

If a is odd & b is even then from property of N-equation we can write c + a is of the form $2\beta^2$ & c + b is of the form α^2
$\Rightarrow$ a ~ b is of the form $\alpha^2 \sim 2\beta^2 \Rightarrow$ if a ~ b = 1 then $\alpha^2 \sim 2\beta^2 = 1$.

Case-I: When $\alpha^2 - 2\beta^2 = 1$.

Say, $\alpha^2 = (2x + 1)^2 \Rightarrow \beta^2 = 2x(x + 1)$ which is possible only for x = 1 i.e. $3^2 - 2.2^2 = 1$
It produces the only relation under 2nd kind N-equation i.e. $20^2 + 21^2 = 29^2$.

Case-II: When $2\beta^2 - \alpha^2 = 1$

Here, $\beta^2 = x^2 + (x + 1)^2$
$\Rightarrow (2x + 1)^2$ will be of the form 'sum of two consecutive integers' square' when
$x^2 + (x + 1)^2$ is a square integer.
We have $3^2 + 4^2 = 5^2$ where x = 3
Hence, next consecutive phenomenon will be observed for $(2.3 + 1)^2 < 50$ i.e. 2.5^2 i.e. for $k = 7^2$.

Say, $(b-1)^2 + b^2 = (b + 7^2)^2 \Rightarrow b = 120$ that follows the relation $119^2 + 120^2 = 169^2$.
Next phenomenon will be observed for $(2.119 + 1)^2 < 2.169^2$ i.e. for $k = 239^2$.
Say, $(b-1)^2 + b^2 = (b + 239^2)^2 \Rightarrow b = 137904$ that follows $137903^2 + 137904^2 = 195025^2$.
Similarly, next (a, b) – consecutive phenomenon will be observed as $183648021599^2 + 183648021600^2 = 259717522849^2$ & so on.

So, condition for this (a, b)-consecutive phenomenon is that the leading set must have the same phenomenon.

2. If $u^2 + 1$ is composite then $u^4 + 1$ is also composite that follows the proof that Fermat Number ($F_n = 2^{2^{\wedge}n} + 1$) always represents a composite number for $n > 4$.

As $u^2 + 1$ is composite it must have another wing say, $u^2 + 1 = a^2 + b^2$
Now $u^4 + 1 = (a^2 + b^2)^2 - 2u^2 = (a^2 - b^2)^2 + (2ab)^2 - 2u^2$.
It clearly indicates $u^4 + 1$ must satisfy the left hand odd element of a 2nd kind N-equation or 1st kind N-equation except $k = 1$ as because $(a^2 - b^2)$ & $(2ab)$ are not two consecutive elements as observed from 1st Fermat composite number i.e. $F_5 = (2^{2^{\wedge}4})^2 + 1 = 20449^2 + 62264^2$ where 20449 & 62264 are not consecutive.
Mode of formation of successive Fermat numbers and that of successive (a, b) consecutive numbers are completely different.
Hence, $u^4 + 1$ cannot be prime. Because all the left hand odd elements of a N-equation except $k = 1$ are composite.

Conclusion:

Now I believe that the proof of this composite nature of all Fermat numbers for $n > 4$ is complete in all respects and will be accepted by all mathematical communities. In general we can conclude that any number in the form of $F_n = (\text{odd})^{2^{\wedge}n} + (\text{even})^{2^{\wedge}n}$ where gcd(odd, even) = 1 produces constantly composite numbers after certain operations of n for prime numbers. So at initial stage if $(\text{odd})^2 + (\text{even})^2$ is found to be composite it will never produce prime numbers. The possibility of constant generation of prime numbers cannot be ruled out also.
I also believe that with the help of this N_s operation and N_d operation as defined in my earlier papers published in 'IJSER, Houston, USA' so many problems in Number Theory are possible to be solved.

References

Books

[1] Academic text books of Algebra.

[2] In August edition, Vol-4 2013 of IJSER, Author: self

[3] In June edition, Vol-6, 2015 of IJSER, Author: self

Author: Debajit Das (dasdebjit@indianoil.in)
(Company: Indian Oil Corporation Ltd, Country: INDIA)

I have already introduced myself in my earlier publications. By profession I am a civil Engineer working in a Public Sector Oil Company as a Senior Project Manager. But to play with mathematics particularly in the field of Number theory is my passion. I am Indian, born and brought up at Kolkata, West Bengal. My date of birth is 12th July, 1958.